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Development of a Granular Media Model for Finite Element Analysis

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Final report

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Preface

The research reported herein was sponsored by the U.S. Army Corps of Engineers through the Research, Development, Testing, and Evaluation (RDT&E) Program, Pavements Research Work Package, AT22. This research was conducted by the U.S. Army Engineer Research and Development Center (ERDC), Geotechnical and Structures Laboratory (GSL), Airfields and Pavements Division (APD), Vicksburg, MS.

The study was conducted under the general supervision of Dr. Michael J. O'Connor, Director, GSL, Dr. W. F. Marcuson III, former Director, GSL, and Dr. David W. Pittman, Chief, APD. Dr. John B. Metcalf, Louisiana State University, provided much insight and technical advise during the testing, analysis, and publication phases of this research. Technical direction was provided by Dr. Raymond S. Rollings, Team Leader, APD, Analytical Modeling Team. Dr. John F. Peters, GSL, provided essential technical guidance and critical developmental expertise throughout all aspects of this project. Dr. Donald M. Smith, APD, was the principal investigator of the project. Drs. Peters, Smith, and Rollings authored this report. Messrs. Tommy Carr, Robert Walker, and David Stinson, ERDC, provided assistance during the testing, programming, and analysis phases of this research.

At the time of publication of this report, Dr. James R. Houston was Director of ERDC, and COL James S. Weller, EN, was Commander.

1   Introduction

Airfield pavement design is a complex blend of relatively simple linear elastic theory, fatigue concepts, empirical relationships derived from small and full-scale tests, and pragmatic adjustments to reflect observations of in-service pavements. This philosophy served the design community well for many years as it allowed total thickness, asphalt concrete pavement thickness, and material requirements for constituent layers in the pavement to be determined to avoid a pre-selected level of distress in the pavement. For airfields, this level of distress at "design" failure was selected to be one inch of shear rutting in the subgrade or fatigue cracking of the asphalt concrete.

However, today's designers are being asked to predict pavement performance. This is a far more complex task than simply providing safe thickness and specifications for the material. To deal with this new challenge, the design community must have material models that predict cumulative deformations under repetitive aircraft loads. With heavy loading, such as may be encountered with many airfields, the nonlinear response of base course materials must be considered when predicting pavement performance. The advances made in computational mechanics have created new tools of application for this type of problem. Theoretically rigorous material models may be implemented within many of the general-purpose finite element computer programs available today. In order to apply these material models, mechanical response data are required to calibrate the necessary model parameters, Barker and Gonzalez (1991).

The flexible pavement used in military airfields commonly consists of a thin asphalt concrete (AC) surface to provide a high-quality waterproof surface, and relatively thick layers of granular base and subbase down to the subgrade. These thick granular layers are used to reduce the stresses applied by aircraft traffic on the pavement surface. A typical pavement of this type is shown in Figure 1 (Webster 1993).

The magnitude and frequency of loading in airfield pavements are very different from typical highway pavements, as shown in Figure 2. The amount of load repetitions applied to airfield pavement is several orders of magnitude less than that seen in highways. A high-volume highway may experience 60 million load repetitions, while a high volume airfield may only experience 50,000 aircraft coverages in a 20-year period. These differences led to a divergence in the research focus between the airfield and highway pavement communities. The major focus of research into highway flexible pavement design has been in the area of viscous fatigue modeling of asphalt concrete. The airfield pavement
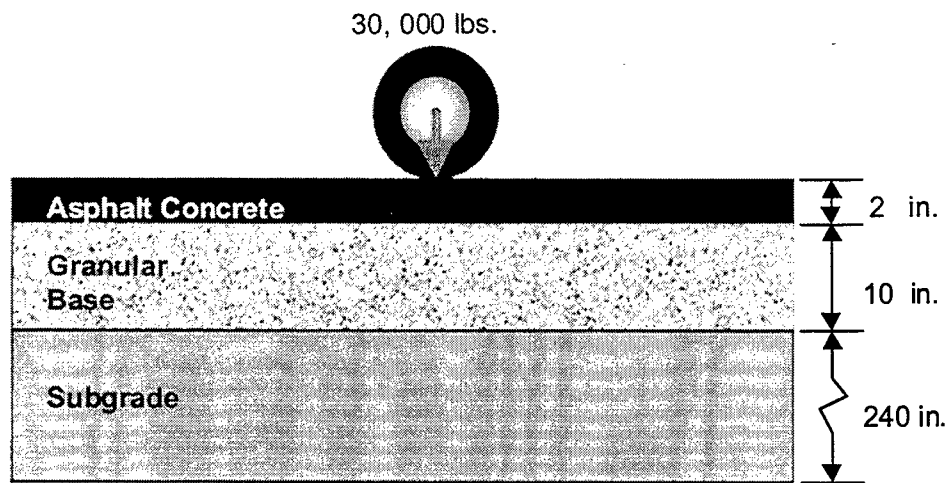
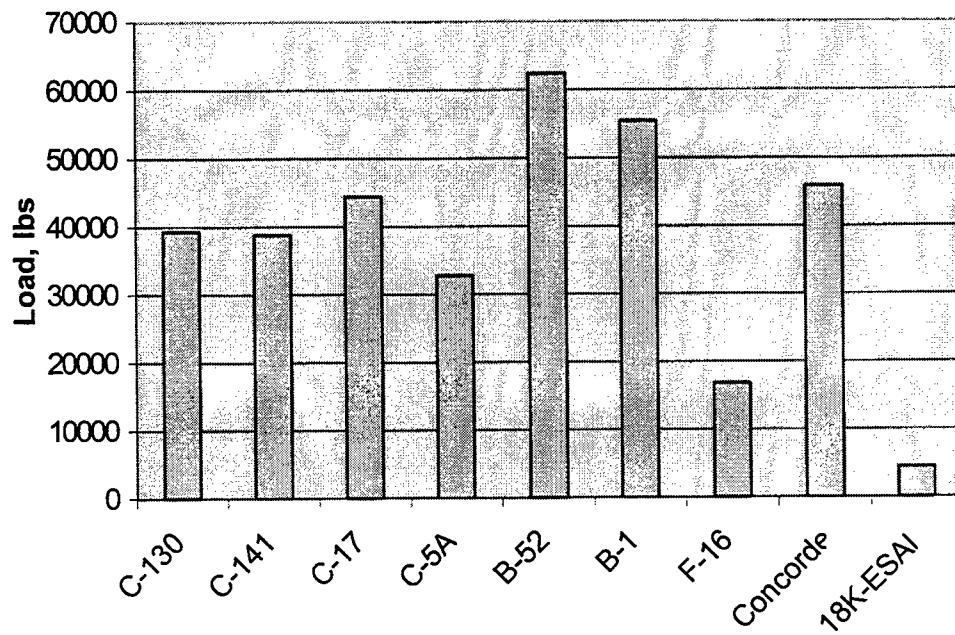Figure 1.   Flexible pavement from Webster (1993)

Figure 2.   Comparison of aircraft single wheel loads and truck tire loads
            (18K-ESAL)

community has been required to broaden the focus of analytical research to include the AC and all supporting layers.

Response predictions for granular bases have always posed one of the most difficult analytical problems in traditional airfield pavement design methodologies. For this reason, the granular layers have never been treated explicitly in design as have the AC layer and subgrade layer, which have used predictive models for cracking in the AC and rutting in the subgrade as a function of linear-elastic strain and material properties. Instead these granular layers were carefully specified in terms of gradation, plasticity, and in situ density to minimize deformation under traffic. However, to eventually develop theoretical methods to predict performance of the pavement, sound methods are needed that predict plastic deformation within these granular layers.

The structural components of flexible pavements are highly nonlinear-elastic plastic materials. With heavy loading, such as may be encountered with many roads and airfields, the nonlinear response of pavement materials should be considered when predicting pavement performance. The advances made in computational mechanics have created new tools, such as the newer generation finite element codes, for this type of problem. The beauty of the finite element method is that it can incorporate both material and handle arbitrary geometry. Theoretically rigorous material models may be implemented within many of the general-purpose finite element computer programs available today. In order to apply these material models, mechanical response data are required to calibrate the necessary model parameters.

2 Model Requirements

Typical rational design procedures couple theoretical response models that predict traffic-induced stresses, strains, and deflections with damage models for fatigue cracking and pavement rutting. The various layers in a pavement system are characterized by their engineering properties and the structural design is subsequently based upon limiting stresses, strains, or deflections computed at certain critical locations in the pavement structure. The procedures use an iterative process, which involves theoretical response analysis, material characterization, distress prediction, and adjustment factors. Several rational (mechanistic) pavement design procedures have been introduced into design over the past years.

The objective of this research is to provide a predictive method for modeling the response of unbounded granular layers in flexible pavements subjected to aircraft loads. The essential features of pavement response that are required from a constitutive model include nonlinear elastic response, permanent or plastic deformation resulting from yield, cyclic loading, strain softening/hardening, and shear-dilatancy. A pavement model should be simple in operation, implementation, and calibration. The model must be executable within a proven general-purpose finite element code such as ABAQUS from HKS, Inc. The model must also provide pavement analysts with the capability of predicting the performance of unbounded materials under traffic loading.

3 Historical Foundations of Model

The model is a product of nearly twenty year's research (1980 to present) at Waterways Experiment Station (WES) and the Engineering Research and Development Center (ERDC) of the U.S. Army Corps of Engineers on constitutive theory for soil in civil engineering applications. This period is marked by enormous interest in constitutive theory for soil by the geotechnical profession in general, during which an abundance of modeling concepts was proposed. The focus of the work at WES and ERDC was to delineate the relationships among the various modeling concepts and the physics of soil behavior. The goal was to produce a constitutive model based on consistent set of mechanical and thermodynamic principles that had predictive capabilities and was suitable for practical numerical analysis. The key ideas for the model can be found in Valanis and Peters (1991) (see also Valanis and Reed 1986, Peters and Valanis 1992) which describes a theory based on endochronic plasticity that is consistent with the critical state theory first proposed Roscoe and his coworkers (1958). An important result of this work was the clear distinction between kinematic hardening, which arises naturally from the internal variable theory of irreversible thermodynamics, and isotropic hardening, which arises naturally from evolution of the independent state variables and past stress history of the material. The theory was applied to a number of modeling applications (e.g. Issa et al. 1995), which demonstrated its potential for predicting mechanical response of a wide range of engineering materials. In addition, independent experimental research was directed at describing the plastic response mechanisms under very generalized loading. In general, the central role of kinematic hardening predicted by the theory seems to be consistent with experimental observations.

Despite its success, the theory of Peters and Valanis (1991) did not fully meet the needs for engineering practice. In particular, realistic endochronic models are difficult to calibrate and implement numerically, as is apparent from the efforts of Issa et al. (1995). The endochronic theory in general requires a facility with mathematical analysis not found in the experimentalists and engineers who are most important to its practical use. A version of the theory was thus crafted that is accessible through intuitive rather than mathematical descriptions without loss in predictive capability or adherence to fundamental physical principles.

The multi-mechanical model achieves the desired simplicity through a particular interpretation of the notion of intrinsic time first introduced by Valanis

(1971, 1980). As described in the next section, by altering the definition of time a strictly mechanistic interpretation of the theory is possible. The definition of time is, of course, fundamental to the theory and the simplicity is gained has physical consequences. Initially, it was hoped that the physical response of the new model would be a sufficiently accurate approximation of the original endochronic model. Upon further examination, it appears that the multi-mechanical model may actually give a better representation of granular media than the original endochronic model.

4   Theoretical Background

The theory of internal variables, as described by Valanis (1971), relates the stress, σ_{ij} to the Helmholtz free energy ψ by

$$\sigma_{ij} = \frac{\partial \psi}{\partial \varepsilon_{ij}}(\varepsilon_{ij}, q_{ij}^{(r)}). \quad\text{...} \quad (1)$$

where ε_{ij} is the strain and $q_{ij}^{(r)}$ are internal variables which capture the irreversibility of the system. For soils application it is convenient to decompose the stress into its shear and hydrostatic components $\sigma_{ij} = s_{ij} + \sigma \delta_{ij}$ such that the free energy is expressed as $\psi = \psi^{s} + \psi^{h}$ with

$$s_{ij} = \frac{\partial \psi^{s}}{\partial e_{ij}}(e_{ij}, q_{ij}^{(r)}). \quad\text{...} \quad (2)$$

and

$$\sigma = \frac{\partial \psi^{h}}{\partial \varepsilon_{H}}(\varepsilon_{H}, q^{(r)}). \quad\text{...} \quad (3)$$

The subscript H refers to the hydrostatic component that differs from the total volumetric strain, ε (Valanis and Peters 1991). Typically, for elastically isotropic materials the free energy is defined as

$$\psi^{s} = \frac{1}{2}\sum_{r=1}^{N_{s}} A_{r} \left\| e_{ij} - q_{ij}^{(r)} \right\|^{2} \quad\text{...} \quad (4)$$

and

$$\psi^{h} = \frac{1}{2}\sum_{r=1}^{N_{h}} B_{r} \left\| \varepsilon_{H} - q^{(r)} \right\|^{2} \quad\text{...} \quad (5)$$

Associated with each internal variable is a thermodynamic force $Q_{ij}^{(r)} = -\partial \psi / \partial \varepsilon_{ij}$ so that

$$Q_{ij}^{(r)} = A_r(e_{ij} - q_{ij}^{(r)}) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (6)$$

$$Q^{(r)} = B_r(\varepsilon_H - q^{(r)}). \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (7)$$

It follows that $s_{ij} = \sum Q_{ij}^{(r)}$ and $\sigma = \sum Q^{(r)}$. The change of thermodynamic shear force is

$$dQ_{ij}^{(r)} = A_r(de_{ij} - dq_{ij}^{(r)}) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (8)$$

The internal variable is removed from Equation 6 through a rate equation that satisfies the thermodynamic restriction $dq_{ij}^{(r)}Q_{ij}^{(r)} \geq 0$. For example, a viscous resistance law would give

$$Q_{ij}^{(r)} = \mu \dot{q}_{ij}^{(r)} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (9)$$

where the dot implies differentiation with respect to time. Substitution of Equation 9 into Equation 8 gives the evolution law for a linear viscoelastic material. Valanis noted that the rate relationships can be cast in incremental form and thus do not depend on the definition of time. In particular, the equations for rate independent plasticity results from defining an *intrinsic* or *endochronic* time based on plastic strain path length. Valanis (1971 and 1980) showed that the plastic response to be expressed as a history integral (kernel functions), similar to viscoelastic materials. Through restrictive assumptions on the character of the kernel functions in the history integrals, classical kinematic hardening plasticity based on a yield surface expressed in terms of back stress results. Introduction of independent state variables into the time scale results in the theory for isotropically hardening plasticity. Importantly, models for combined kinematic and isotropic hardening evolved from fundamental assumptions without the need for complex ad hoc rules for expansion and translation of the yield surface.

For soils, Valanis and Peters (1991) added a coupling between hydrostatic and shear mechanisms that captured correctly volumetric strains produced by cyclic loading. In this coupling the hydrostatic strain, $d\varepsilon_H$, differs from the measured volumetric strain, $d\varepsilon$, by a coupling strain, $d\varepsilon_c$, using an incremental relation, $d\varepsilon_H = d\varepsilon - d\varepsilon_c$. The coupling strain increment is proportional to the increment of plastic shear strain through a stress-dilatancy relationship similar to that of critical state soil mechanics. The coupling laws in the two theories differ; the endochronic theory includes a provision for load reversals, which is a feature not considered in the critical state theory.

The multi-mechanical model arises from a particular definition of intrinsic time. Valanis (1971) notes that the time is not only intrinsic to the material, but it could be intrinsic to the mechanism. Thus, each mechanism could have an *autonomous* time. The consequence of using autonomous time is that, in the absence of isotropic hardening, hysteresis loops are closed. Endochronic time, which couples all mechanisms, produces hysteresis loops that do not close completely. Cyclic loading in real materials produce hysteresis loops that do not

close, indicating the endochronic time gives the better representation of cyclic behavior. As is discussed in the section on Model Description, the lack of closure in hysteresis loops for soils appears to be related to the coupling effects of Coulomb friction even when autonomous time is used. More to the point, autonomous time yields significant simplification to the theory, which is justification of its use in any case.

By assuming that each mechanism has its own time measure, the model reduces to simple elastic-plastic elements acting in parallel (Figure 3). The strain is common to all mechanisms making integration of the evolution equation a simple matter of independently applying the elastic-plastic yield relationship to each mechanism. The calculation can be done by established numerical techniques for plastic materials. It should be noted that this is not only significantly simpler than previous endochronic laws, but the model is more straightforward than other kinematic hardening laws based on nested yield surfaces or complex translation rules.

The operation of a four-mechanism model in a triaxial test is shown in Figure 4, which illustrates the yield behavior of each mechanism and the stress strain response that results. The elastic-plastic-perfectly-plastic elements act in parallel by making the total strain common to all mechanisms. The total stress is the sum of the component stresses. The secant modulus of the material is equal to the sum of non-failed mechanisms. Thus as the first mechanism fails at point 1 the total stiffness is reduced because the stress in mechanism 1 lies on its yield surface. At points 2 and 3, the second and third mechanisms yield, respectively. Mechanism 4 remains elastic throughout the simulation. At point 4 the load is reversed making all four mechanisms respond elastically. At point 5 the first mechanism yields in extension even though the total stress state is compression. At point 6 the second mechanism yields in extension. The remaining two mechanisms remain elastic. At the next load reversal, point 7, all four mechanisms respond elastically up to point 8, where mechanism 1 again yields in compression. At this juncture a critical aspect of frictional systems is observed. When the compression yield points 1 and 8 are compared, the mean stress at point 8 is seen to be lower than point 1. As a result, the shear stress at yield for point 1 is higher than it is at point 8. A similar relationship between yield points at points 2 and 9 is seen for the second mechanism. The net result is that the hysteresis loop does not close, causing a ratcheting to occur under cyclic loading. The situation is magnified when the volume change is restricted, as in the case of saturated soil without drainage, because the fluctuations in mean stress can be quite large. For example, the liquefaction instability during cyclic loading, which is predicted by the model, is due to this effect. The ratcheting under cyclic shear loading is the product of several interacting mechanisms for which the hydrostatic stress acts like a varying parameter. Ratcheting is not a prescribed feature of the shear response; it is a predicted effect.
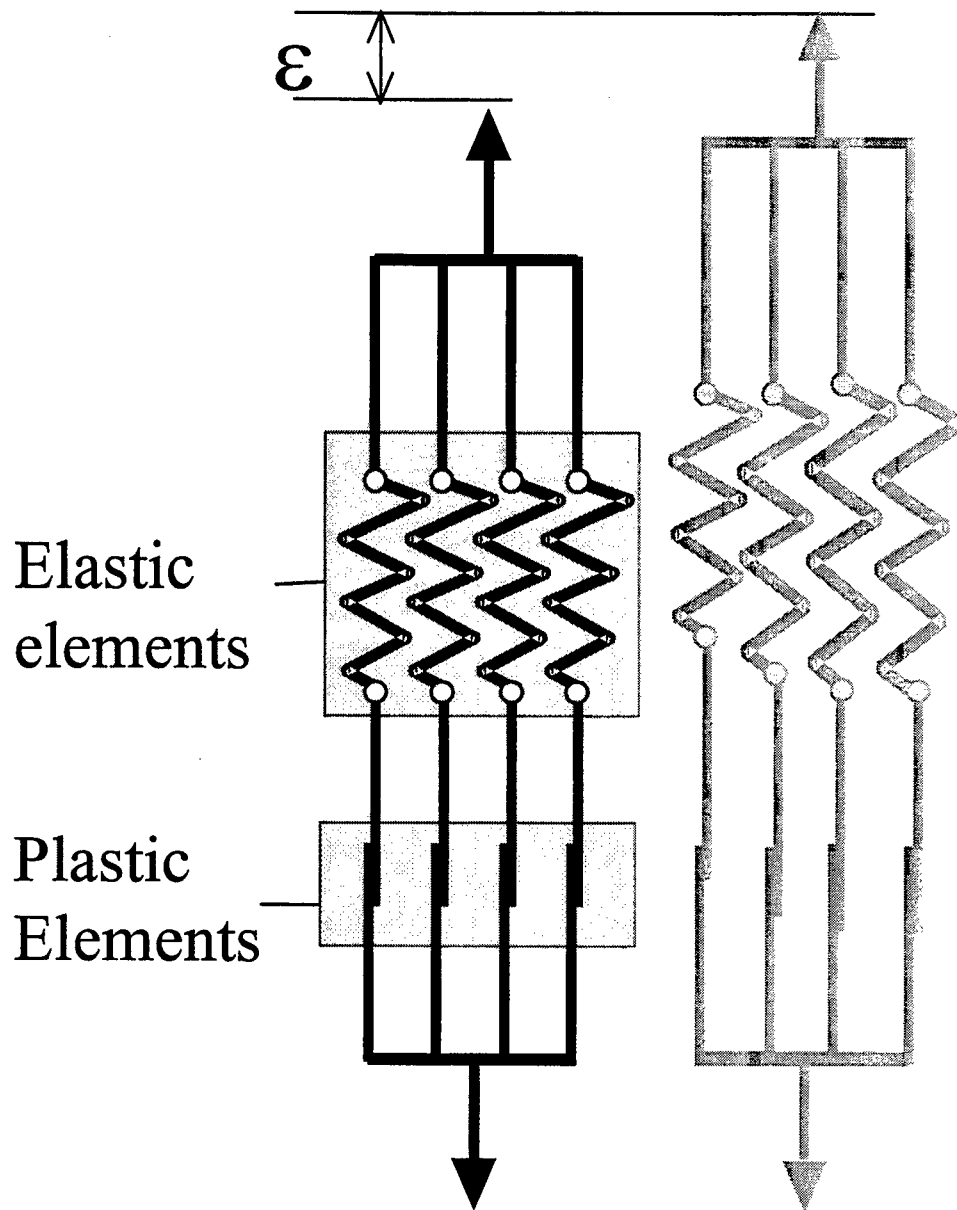
Figure 3.   Schematic representation of plasticity response model. The combined
simple responses of elastic-perfectly plastic elements produce a
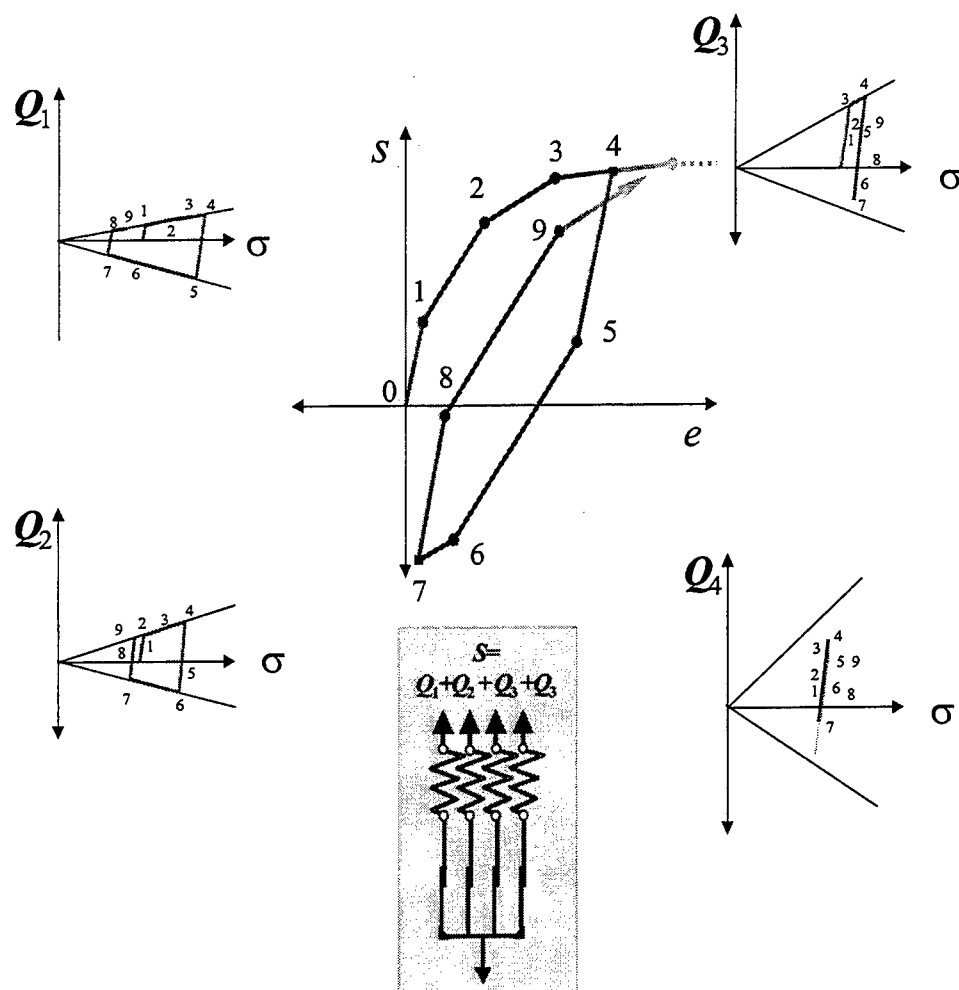complex kinematic hardening response

Figure 4. Response to cyclic loading created by interaction of parallel elastic-
perfectly plastic components

5   Model Calibration

Smith (2000) presents an application of the WES model using four mechanisms. The calibration was based on a suite of drained triaxial loading tests that included both monotonic and cyclic loading components. Although only the monotonic loading tests are required for calibration, the data from cyclic loading proved to be essential for an accurate calibration.

Calibration from Monotonic Loading Tests

The procedure for calibrating the model requires a set of several triaxial tests, either drained or undrained, with pore pressure measurements. First, the relation between mean effective stress and void ratio at the ultimate state is plotted similar to an e - log p curve as shown on Figure 5. The slope C_c and intercept, e_{ref}, are used to determine the relation between void ratio and the reference pressure, P_e. Next, the hydrostatic stress-strain curve is plotted in a normalized form in which the hydrostatic stress is divided by the reference stress. In this form the hardening effect of void ratio decrease is removed, leaving the fundamental curve. The normalized curve is then divided into regions to be represented by each mechanism. The yield stress associated with each mechanism is thus determined. The stiffness of each mechanism is determined by the change in modulus that occurs as each yield limit is crossed.

A similar procedure is carried out for the shear response. The shear yield limit is determined for each mechanism. Friction angles are selected based on the ultimate friction angle at a stress level close to that of the expected service loads. From these data, the distribution factor for hydrostatic stress can be determined for each mechanism. The calibration for the shear modulus the same as that for the bulk modulus of the hydrostatic mechanism.

Repeated Loading

The predictions made using the calibration presented by Smith (2000) produced cyclic behavior that was somewhat different form the laboratory tests. The model did produce hysteresis, but the shape and size of the hysteresis loops at low strain levels and the magnitude of the strain at which the cyclic behavior
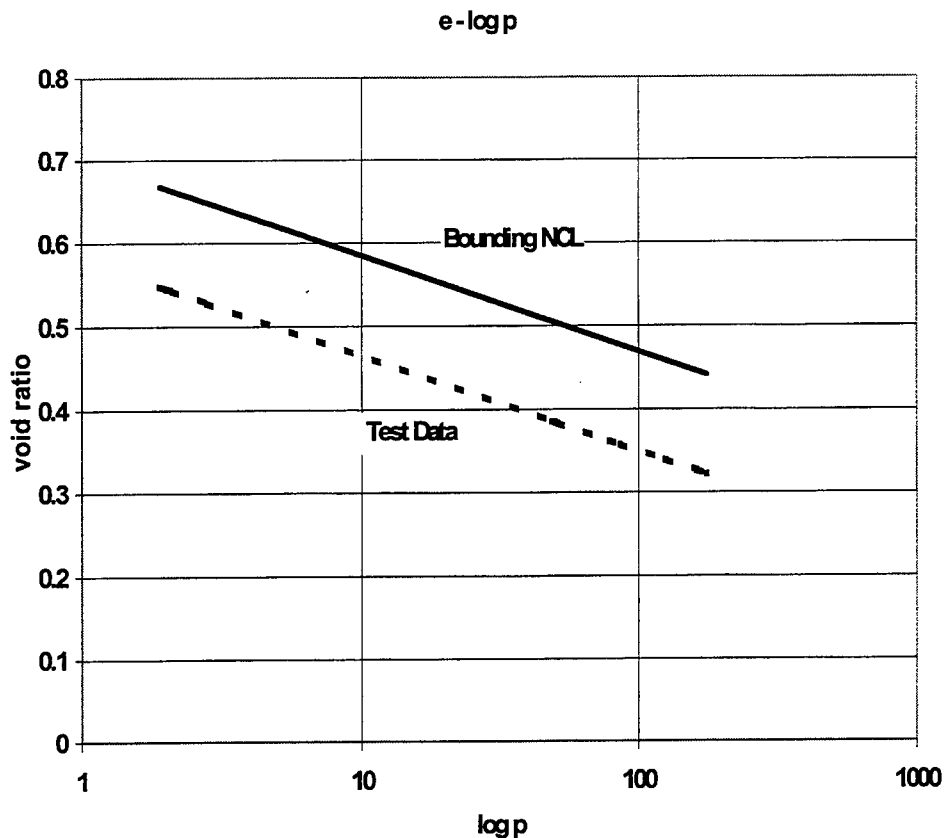
e-log p

Figure 5.   Void ratio versus log normal stress plot used to determine NCL for
WES MM Model

began was different from the test data as shown in Figure 6. This illustrates an important aspect of the calibration process. Although the model can be completely calibrated from a test with a monotonically increasing loads, the cyclic behavior may not be represented well. The stress-strain data from the initial part of the monotonic loading are often not representative of the initial yield behavior that may dominate cyclic response. Also, seemingly subtle differences in the initial loading may make large differences in the predicted hysteresis of the cyclic response. Finally, the virtually continuous stress-strain response is approximated by a small number of mechanisms in the model. An accurate depiction of hysteresis during cycling at small strain may require assigning more mechanisms to the initial part of the curve, possibly at the expense of detail in the yield near failure. Alternatively, a greater number of mechanisms could be used.

A modified calibration was completed (shown in Table 1 and Table 2), and the cyclic test was rerun with the new calibration. The ability of the WES MM model to closely capture cyclic response was clearly demonstrated in this analysis. However, obtaining this calibration required numerous iterations and intimate knowledge of model behavior. One of the original goals of this model development was to produce a constitutive model that would be relatively easy

Crushed Limestone Type 610
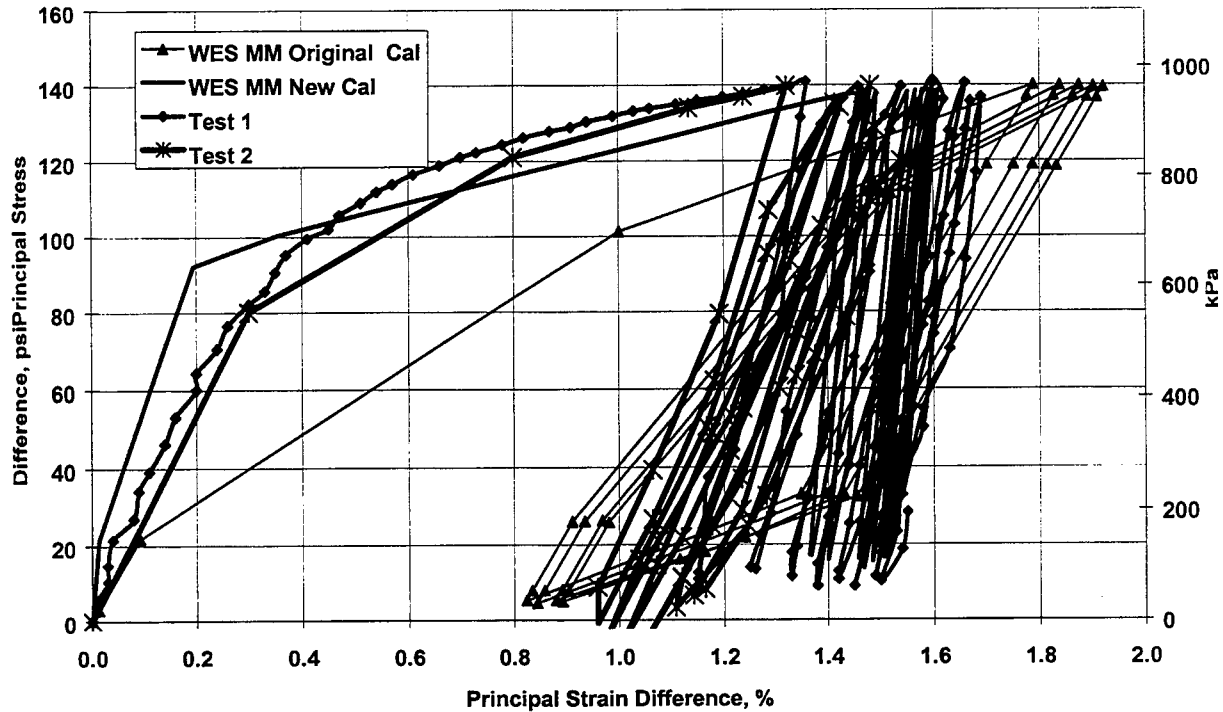**Triaxial Compression at 50 psi (344.7 kPa)**

Figure 6. Comparison of FEM prediction of cyclic response to test data for two model calibrations

| Table 1 |
| **Global Properties for Modified Calibration** |

Property	Magnitude
Friction Angle	48 degrees
Cohesion	0.25 psi (1.72 kPa)
Bulk Modulus	100000 psi (689.5 MPa)
Shear Modulus	200000 psi (689.5 MPa)
Phi Ratio	0.50
Hydrostatic Intercept	0.70 psi (4.82 kPa)
Reciprocal of Cc	8.685
Shear-volume Factor	0.72
OC factor	1.80
Dilatancy Rate Factor	1.00

Table 2
Mechanism Properties for Modified Calibration

	Mechanism			
	1	2	3	4
Phi Fraction	0.1	0.25	0.6	0.9
Mean Stress Fraction	2.2	0.86	0.3	0.35
Shear Stiffness Distribution	0.49	0.26	0.068	0.011
Compression Limit	0.018	0.9	1.00	1.00
Volumetric Stiffness Distribution	0.565	0.38	0.02	0.035

to calibrate from standard geotechnical laboratory tests and produce reasonable predictions of pavement response to load. Analysis of the model behavior is presently underway to identify better the effect that each parameter has on stress strain response and the relationship between number of mechanisms and range of accuracy of the calibration. Preliminary results of this effort indicate that the trial and error can be completely removed from the calibration procedure.

This modified calibration also proved to create numerical convergence problems with ABAQUS when applied to the field test section FEM analyses. When using a commercial code like ABAQUS, one does not have access to the source code for the FEM program. As a result, when problems with convergence are encountered and can not be solved through the use of interation time-stepping options, other avenues of completing an analysis, such as equivalent alternate material model calibrations, must be considered. Even though the modified cyclic calibration produces excellent stress-strain agreement with the test data at low stress levels, the amount of permanent strain accumulated from each cycle is very close for both calibrations. Given these considerations, the original standard calibration was used for the test section analyses.

6 Computational Example: ABAQUS FEM Analysis of Test Section

The ABAQUS FEM code was used to analyze the response of a pavement test section Webster (1993) shown in Figure 1. All ABAQUS computations were conducted on SGI ORIGIN 2000 supercomputers. Finite element model development for ABAQUS was accomplished interactively on engineering workstations using the MSC Software Corporation's PATRAN software incorporating an ABAQUS application interface. MSC/PATRAN was also used to post-process many of the results from ABAQUS. A 2-D static axisymmetric analysis was performed using the WES multi-mechanical model for the base course and linear elastic properties for the asphalt and subgrade layers. The purpose for this analysis was to demonstrate the ability to predict permanent deformation in a granular pavement layer.

The information available from the test section data did not allow for direct calibration of the asphalt and subgrade properties. However, there was enough information to arrive at reasonable values for the elastic constants for the asphalt and subgrade layers. The Young's modulus, E, was 500,000 psi (3447.5 MPa) for the asphalt layer and 18,000 psi (124.1 MPa) for the CH clay subgrade. A Poisson's ratio of 0.35 was used for both layers. Typical values for these material constants can be found in a number of sources (Ulidtz 1998). The deflection values obtained from multidepth deflectometers (MDD) were used to backcalculate these constants to produce reasonable values of deformation under load. In essence the MDD reading under load enabled a crude back calculation of elastic constants to be performed, thus enabling the base course layer to see a stress state very similar to the true state of stress under load. The values used for these elastic constants in the subgrade material represent more than just a material property. They are an effective foundation stiffness that allows for the subgrade material, lower supporting layers and far boundaries to be included in a simplified system model. The crushed limestone aggregate base course was modeled with the WES multi-mechanical model.

The pavement was modeled using standard 4-node quadrilateral axisymmetric elements from the ABAQUS element library. To correctly model the modified C-130 tire, the load was simulated with a surface pressure of 68 psi (468.8 kPa) applied over a circular area of 442 square inches (0.29 m²) to

produce a total load of 30,000 lb (133 kN). The asphalt layer had a depth of 2 in. (50.8 mm). The granular base was 10 in. (25.4 mm) thick. The total depth of the subgrade was 240 in. (6096 mm) yielding a total model depth of not less than 20 ft (6.25 m).

The test sections were subjected to 5 cycles of a simulated single C-130 tire load. The base course layer was modeled with the WES multi-mechanical model, while the remaining layers were modeled as a linear elastic material. The results of these analyses were compared to MDD measurements from the field test sections to provide model validation and assessment.

Figures 7a shows the deformed shape of the section under the first load application, while Figure 7b shows the deformed shape of the section after the first load was removed. Figure 7c shows the deformed shape of the section under the fifth load application. Figure 7d shows the deformed shape after removal of the fifth load. The value of deformation at the top of the base course was 173 mils (4.39 mm). Webster (1993) reported the value of deformation at the top of the base course in Lane 1-1 to be 165 mils (4.19 mm) under the fifth load application. The predicted deformation under load at the top of the sub-grade was 113 mils (2.87 mm) as compared with a field value of 125 mils (3.17 mm). The agreement between these values verifies the relative accuracy of the overall system calibration as shown in Figure 8.

To validate the ability of the WES MM model to predict plastic accumulated strain under repeated loads, the value of permanent deformation after removal of the load at the top of the base course, 33 mils (0.83 mm), was determined from the analysis and compared with the field value of 40 mils (1.02 mm). The agreement between the FEM predictions and the field measurements are very close.
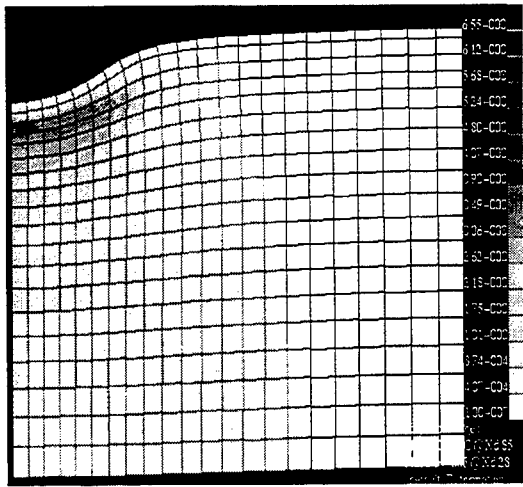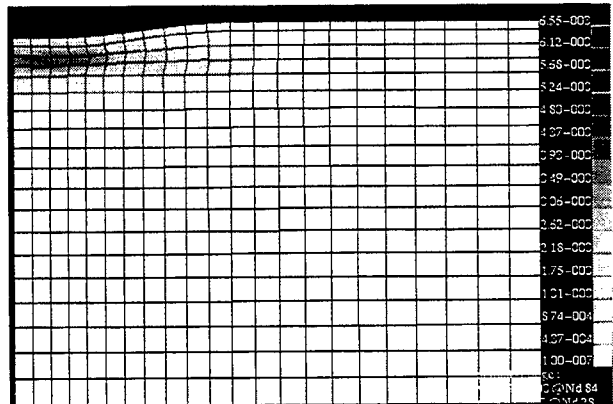
a. Under load cycle 1
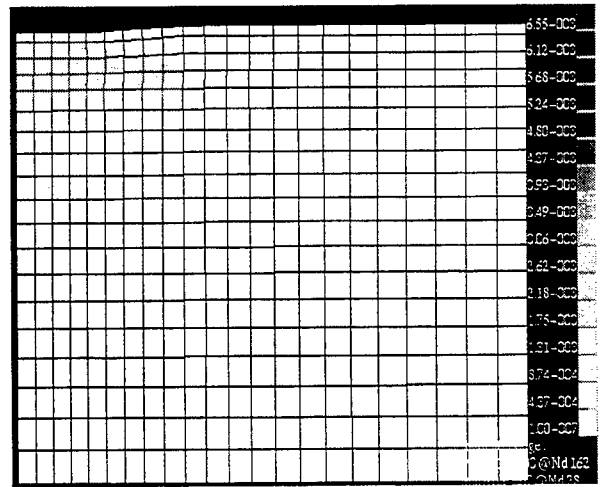
b. After load cycle 1
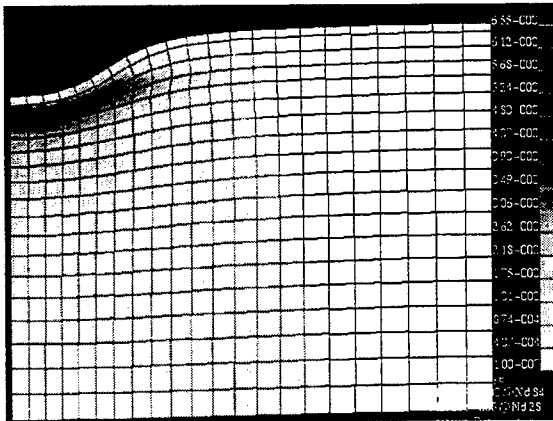
c. Under load cycle 5

d. After load cycle 5

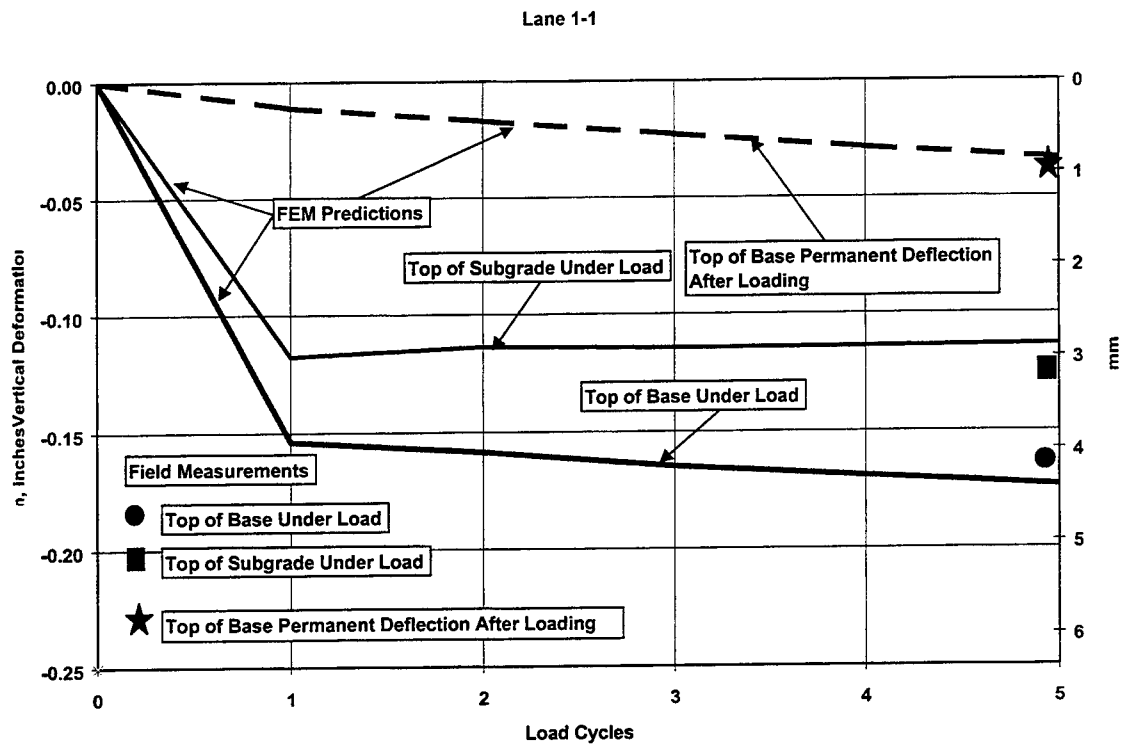Figure 7.   FEM predictions of test section response

Figure 8.   Vertical deformation versus load cycles from FEM simulation

7 Concluding Remarks

This report describes a constitutive model for granular media that is suitable for finite element computations of pavement sub-base, subgrade, and similar materials. The model has demonstrated applicability in the prediction of cumulative permanent deformation of a granular base over a small number of severe load cycles. The model formulation is applicable for a wide range of materials; seemingly diverse materials such as clay subgrade and granular sub-base are distinguished only by values of model parameters. The theoretical concepts upon which the model is based are general and offer an avenue to extend to model to other materials. The following are considered to be critical items to be added in the future to enhance the model's overall applicability in pavements analysis:

a. A mechanism will be added to account for a viscous bituminous binder phase to model asphalt. In this approach, the granular phase represented by the multi-mechanism model accounts for the nonlinearity of the asphalt. The binder phase accounts for rate-dependent response and inhibits volume change.

b. The mechanics of partial saturation are under investigation to determine the interaction of the air-water interface (contractile skin) and the granular matrix. At present, the strengthening effects of partial saturation are accounted in an ad hoc fashion with the multi-mechanical through the cohesion term.

c. A damage law must be added to account for brittle behavior in rigid pavements. The plastic response of concrete can be modeled by the present model owing to its association to the endochronic model (Valanis and Reed 1986).

d. The three-dimensional response of the model will be investigated in detail using cubical and directional shear experiments for sand that were designed especially to study kinematic hardening in granular media (see Alawaji et al. 1990).

The individual physical process addressed by the multi-mechanical model and its extensions are complicated and only partially understood. Yet, each process has essential features that can be modeled in a relatively simple manner. The art of modeling pavement performance rests on the ability to recognize which features are critical to the system behavior. Thus, the success of the system's

approach to pavement modeling depends on combining the various types of response in a robust way. The multi-mechanical model provides a framework to incorporate the key physical phenomenon of elasticity, plasticity, multi-phase behavior, and damage.

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14. ABSTRACT

Today's pavement design calls for more emphasis on pavement performance prediction. This is a far more complex task than traditional designs that strive to provide safe thickness and specifications for the material. Modern computational mechanics provide application tools to deal with this new challenge, provided material models are created that predict cumulative deformations under traffic loading. This paper describes an ongoing research effort to create material models for granular base course material that may be implemented within general-purpose finite element computer programs. The model was evolved from a mechanistic interpretation of endochronic theory, which creates a modular system that can accommodate plasticity, visco-elasticity/plasticity, and damage within a single modeling format. The performance of the model is illustrated with the analysis of permanent deformations in a flexible pavement caused by repeated tire loads.

15. SUBJECT TERMS

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